B.Sc. Part–I Semester—I Examination
MATHEMATICS
(Differential & Integral Calculus)
Paper–II

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt once.

(2) Attempt one question from each unit.

1. Choose the correct alternatives :—

(1) Closed interval [a, b] is defined as :

(a) $\{x / a \le x \le b\}$ (b) $\{x / a < x < b\}$

(c) $\{x / a \le x < b\}$ (d) $\{x / a < x \le b\}$

(2) The value of $\lim_{x \to 0} \frac{\sin x}{x}$ is :

(a) 0 (b) 1

(c) π (d) ∞

(3) The function f(x) has the removable discontinuity if :

(a) $f(x^+) \ne f(x^-)$ (b) $f(x^+) = f(x^-)$

(c) $f(x^+)$, $f(x^-)$ does not exist (d) None

(4) If $y = (2x - 3)^4$ then y_3 is :

(a) 192 (b) $(2x - 3)$

(c) $192(2x - 3)$ (d) 0

(5) $\frac{d}{dx}(\cot^{-1} x)$ is :

(a) $\frac{1}{1+x^2}$ (b) $\frac{1}{1-x^2}$

(c) $\frac{1}{x^2-1}$ (d) $-\frac{1}{1+x^2}$

(6) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ...$ is expansion of :

(a) sin x (b) cos x

(c) e^x (d) tan x

(7) The degree of the homogeneous equation $f(x, y) = \frac{x^3 - y^3}{x - y}$ is :

(a) 2 (b) 3

(c) 1 (d) 0

(8) If u = f(x, y, z) is a homogeneous function of degree n then :

(a) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = z\frac{\partial u}{\partial z}$ (b) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nz$

(c) $y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$ (d) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$

(9) The process of finding the length of arc of a curve by definite integral is known as :

(a) Quadrature (b) Unification

(c) Rectification (d) None

(10) $\int_0^{\pi/8} \cos^3 4x \, dx$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$ 10×1=10

UNIT—I

2. (a) Prove that $\lim_{x \to 2} f(x) = 7$ where f(x) = 2x + 3 and $x \in [0, 5]$. 3

(b) If $\lim_{x \to x_0} = \ell$ then prove that f is bounded on some deleted neighbourhood of x_0. 4

(c) Using $\in - \delta$ definition, prove that f(x) = sin x is continuous for all real values of x. 3

3. (p) Define limit of a function at a point and prove that limit of a function if it exist is unique. 4

(q) Let $f(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

Show that f(x) has removable discontinuity at x = 0. 3

(r) Prove that $\lim_{x \to a} \sin x = \sin a$ and find δ in terms of $\in$. 3

UNIT—II

4. (a) If a function f(x) is differentiable at $x = x_0$ then prove that it is continuous at x_0. 4

(b) If $y = \ell_n(ax + b)$ find y_n. 3

(c) Evaluate $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^2 \tan x}$. 3

5. (p) State and prove Leibnitz theorem. 4

(q) Evaluate $\lim\limits_{x \to 0} \dfrac{1-\cos(x^2)}{x^2 \sin(x^2)}$. 3

(r) If $y = (x + \sqrt{1+x^2})^m$ prove that $(1 + x^2)y_2 + xy_1 - m^2y = 0$. 3

UNIT—III

6. (a) State and prove Rolle's theorem. 4

(b) Discuss the applicability of Lagrange's mean value theorem to the function

$f(x) = Ax^2 + Bx + c$ in [a, b] 3

(c) Prove that $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - ....$ 3

7. (p) State and prove Lagrange's mean value theorem. 4

(q) Verify Cauchy's mean value theorem for

$f(x) = e^x$, $g(x) = e^{-x}$ in [a, b]. 3

(r) Prove that :

$$\ell n(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$$ 3

UNIT—IV

8. (a) If $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2 + y^2 + z^2 \neq 0$

show that $u_{xx} + u_{yy} + u_{zz} = 0$. 3

(b) If u = f(x, y) is a homogeneous differentiable function of degree n in x, y then prove that $xu_x + yu_y = nu$. 4

(c) If u = f(x + ay) + g(x – ay), then show that $u_{yy} = 9^2u_{xx}$. 3

9. (p) If z = f(x, y) and $x = \gamma \cos \theta$, $y = \gamma \sin \theta$ then show that :

$$(Z_\gamma)^2 + \frac{1}{\gamma^2}(Z_\theta)^2 = (Z_x)^2 + (Z_y)^2 .$$ 3

(q) If $u = \operatorname{cosec}^{-1}\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$, then show that

$$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{2}\right)$$ 4

(r) If $Z = f(x^2 - y^2)$ then show that $yZ_x + xZ_y = 0$. 3

UNIT—V

10. (a) Integrate $\int \frac{2x+5}{\sqrt{x^2+3x+1}}dx$. 3

(b) Prove that

$$\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \frac{n-1}{n}\frac{n-3}{n-2}\cdots\cdots \frac{3}{4}\ \frac{1}{2}\ \frac{\pi}{2}, \quad n \text{ is even}$$

$$= \frac{n-1}{n}, \frac{n-3}{n-2} \cdots\cdot \frac{2}{3}, \ n \text{ is odd.}$$ 4

(c) Calculate area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 3

11. (p) Find the length of the arc of the parabola $x^2 = 4ay$ from the vertex to an extremity of the latus-rectum. 3

(q) If $I_n = \int \sin^n x\, dx$ then prove that

$$I_n = -\frac{1}{n}\sin^{n-1} x \cos x \frac{n-1}{n} I_{n-2} .$$ 4

(r) Evaluate $\int_0^1 \frac{1-4x+2x^2}{\sqrt{2x-x^2}}dx$. 3